Stopcock e was then opened, and air was admitted When the reading on the manometer reached a predetermined level, stopcock e was closed, and propane was admitted by opening stopcock d When the pressure in the combustion tube reached atmospheric pressure, stopcock d was closed Stopcock c was then closed, and stopcock f was opened The fuel and air were mixed by raising and lowering the mixing funnel for at least 15 min The eddies that were generated in the mixing process were allowed about 5 min to settle The seal at the bottom of the combustion tube was removed, and a naked flame (1 in long) from a microburner was inserted in the lower end of the tube In the third set of experiments (case III), at the moment of ignition, a spherical, front-surfaced mirror was placed about $\frac{1}{2}$ in below the lower end of the tube The richest fuel mixture for which a flame propagates uniformly throughout the entire length of the tube was used to define the upper limit of inflammability Each set of experiments was conducted several times in order to insure reproducible results

Results

The results of this study may be summarized as follows:

Case I: The upper limit of inflammability is 9 50 This is in agreement with the results of Coward and Jones,[5] who also obtained 9 50 for propane-air Since the apparatus used in this test is similar to Coward and Jones' apparatus, these results established confidence in the test procedures

Case II The upper limit of inflammability is 9 83 This upper limit is slightly (3 4%) above that found in case I

Case III: The upper limit of inflammability is 10 5 This value is 10% greater than that of case I

Thus the following may be concluded:

1) Radiation losses have a significant effect on the determination of the upper limit of inflammability for flames that have carbon particles present in their products of combustion (such as propane)

2) Refer to Fig 2 In case I, nearly all of the radiation escapes from the mixture Part of the radiation passes out the open end of the tube, part passes through the pyrex walls, and part is absorbed by the Pyrex walls and reradiated at different wavelengths and in all directions Only a small portion of this radiation is absorbed by the flame In case II, nearly all of the radiation that is reflected by the silver-coated combustion tube is lost through the ends of the tube In case III, nearly all of the radiation from the flame is reflected back and forth within the combustion tube and makes many passes through the flame front The hot carbon particles in the flame are responsible for most of the radiation and likewise are responsible for most of the absorption of radiation

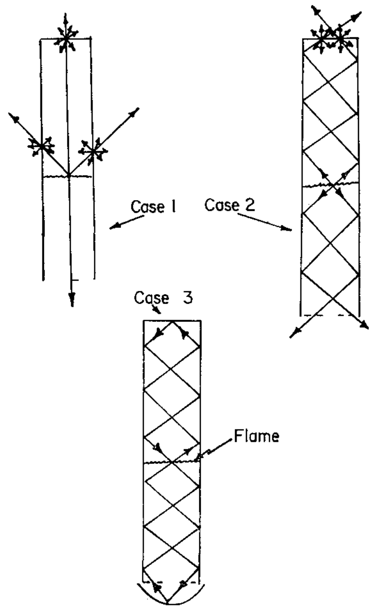

Fig 2 Schematic drawing of radiation from flame

3) The standard inflammability test-apparatus has been designed so that conduction losses to the walls are minimized (diameter of the combustion tube must be at least 2 in), but no provision is made to reduce radiation losses An experimental setup such as described under case III does minimize radiation losses and hence gives a truer measure of the absolute upper limits of inflammability

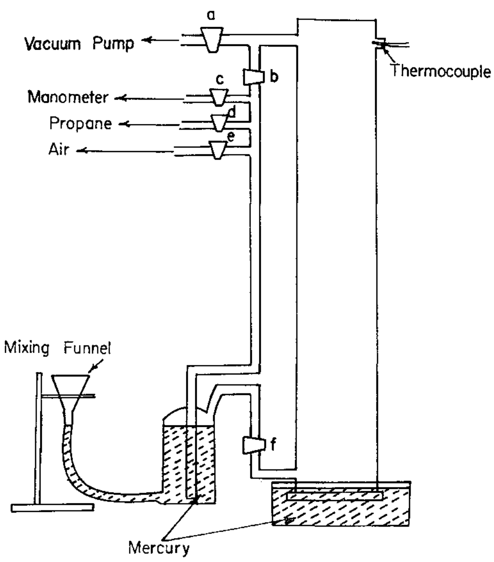

Fig 1 Schematic drawing of apparatus

References

[1] Spalding, D B , "A theory of inflammability limits and flame quenching," Proc Roy Soc (London) **2402A**, 83 (1957)

[2] Mayer, E , "A theory of flames propagation limits due to heat loss," Combust Flame **1**, 438–451 (1957)

[3] Egerton, A and Powling, J , "The limits of flame propagation at atmospheric pressure," Proc Roy Soc (London) **193A**, 172 (1948)

[4] Burgoyne, J H and Williams-Leir, G , "The influence of incombustible vapours on the limits of inflammability of gases and vapours in air," Proc Roy Soc (London) **193A**, 525 (1948)

[5] Coward, H F and Jones, G W , "Limits of inflammability of gases and vapors," U S Bur Mines Rept of Investigation 3966 (1946)

Flow of Viscoelastic Maxwell Fluid

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Introduction

FLOWS of Newtonian viscous fluid under exponential pressure gradient superposed on steady Poiseuille flow through coaxial cylinders have been discussed by Verma [1] Here we have considered the flow of Maxwell fluid under exponentially decreasing pressure gradient superposed on the steady laminar flow of the same fluid between two coaxial circular cylinders

We know that a viscoelastic fluid of the Maxwell type (i e , a spring and a dash-pot arranged in series) is governed by

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the equations

$$\left.\begin{aligned}\tau_{ij} &= -p\delta_{ij} + \tau'_{ij}\\ [1 + x(\partial/\partial t)]\tau'_{ij} &= 2\mu e_{ij}\\ e_{ij} &= \tfrac{1}{2}(v_{i\,j} + v_{j\,i})\end{aligned}\right\} \qquad (1)$$

where τ'_{ij} is the deviatoric stress tensor, p is the pressure, and x, μ are material constants (x being of the dimension of time is called the relaxation time and μ is the viscous parameter) and v_i $(i = 1, 2, 3)$ are the velocity components

Let us choose cylindrical polar coordinates (r, θ, z) with z axis along the axis of the cylinders If the flow direction is assumed to be parallel to the z axis, the velocity components will be given by

$$u = 0 \qquad v = 0 \qquad w = w(r,t) \qquad (2)$$

The equations of motion give, with the help of (1),

$$\left.\begin{aligned}\left(1 + x\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial t} &= -\frac{1}{\rho}\left(1 + x\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right)\\ \nu &= \mu/\rho\\ 0 &= -(1/\rho)(\partial p/\partial r)\end{aligned}\right\} \qquad (3)$$

From the second equation of (3) we see that p becomes a function of z and t

Let us assume that

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = c_0 + \sum_{n=1}^{\infty} c_n e^{-nt} \qquad (4)$$

and

$$w = w_0 + \sum_{n=1}^{\infty} w_n e^{-nt} \qquad (5)$$

where c_n and w_n are real and w_n is a function of r only

Substituting for w and $-(1/\rho)(dp/dz)$ in (3), and comparing terms of the same family, we get

$$\frac{d^2 w_0}{dr^2} + \frac{1}{r}\frac{dw_0}{dr} + \frac{c_0}{\nu} = 0 \qquad (6)$$

and

$$\frac{d^2 w_n}{dr^2} + \frac{1}{r}\frac{dw_n}{dr} + \frac{c_n + nw_n}{\nu}(1 - nx) = 0 \qquad (7)$$

Integrating (6), we get

$$w_0 = -(c_0 r^2/4\nu) + A \log r + B \qquad (8)$$

In (7) let us put

$$(c_n + nw_n)/\nu = V$$

Then we get

$$\frac{d^2 V}{dr^2} + \frac{1}{r}\frac{dV}{dr} + \frac{n(1 - nx)}{\nu} V = 0$$

the solution of which is

$$V = D_n J_0(Kr) + E_n Y_0(Kr)$$

where

$$K = \left[\frac{n(1 - nx)}{\nu}\right]^{1/2}$$

and J_0 and Y_0 are Bessel functions of the first and second kinds of zero order

Hence, the general expression for w is given by

$$w = -\frac{c_0 r^2}{4\nu} + A \log r + B - \sum_{n=1}^{\infty}\frac{c_n}{n}e^{-nt} + \sum_{n=1}^{\infty}\frac{1}{n}[D_n J_0(Kr) + E_n Y_0(Kr)]e^{-nt}$$

Using the boundary conditions $w = 0$ on $r = a$ and on $r = b$, and making use of the condition that, before superposing the exponential motion, the motion is steady and laminar, we can evaluate the unknown constants A, B, D_n, and E_n

Hence, we get

$$w = -\frac{c_0}{4\nu}\left[(r^2 - a^2) - (b^2 - a^2)\frac{\log(r/a)}{\log(b/a)}\right] + \sum_{n=1}^{\infty}\left\{\frac{J_0(Kr)[Y_0(Kb) - Y_0(Ka)] - Y_0(Kr)[J_0(Kb) - J_0(Ka)]}{J_0(Ka)Y_0(Kb) - J_0(Kb)Y_0(Ka)} - 1\right\}\frac{c_n}{n}e^{-nt} \qquad (9)$$

When K is small, we have

$$J_0(Kr) \simeq 1 - (r^2K^2/4)$$

and

$$Y_0(Kr) \simeq \frac{2}{\pi}\left[\left(\gamma + \log\frac{Kr}{2}\right)\left(1 - \frac{K^2r^2}{4}\right) + \frac{K^2r^2}{4}\right]$$

where γ is Euler's constant Hence,

$$J_0(Kr)[Y_0(Kb) - Y_0(Ka)] - Y_0(Kr)[J_0(Kb) - J_0(Ka)] \simeq \frac{2}{\pi}\left\{\log\frac{b}{a} - \frac{K^2}{4}\left[r^2\log\frac{b}{a} + b^2\log\frac{Kb}{2} - a^2\log\frac{Ka}{2} - (b^2 - a^2)\left(1 + \log\frac{Kr}{2}\right)\right]\right\}$$

and

$$J_0(Ka)Y_0(Kb) - J_0(Kb)Y_0(Ka) \simeq \frac{2}{\pi}\left\{\log\frac{b}{a} - \frac{K^2}{4}\left[(a^2 + b^2)\log\frac{b}{a} - (b^2 - a^2)\right]\right\}$$

Substituting in (9) we have

$$\begin{aligned}w &= -\frac{c_0}{4\nu}\left[(r^2 - a^2) - (b^2 - a^2)\frac{\log(r/a)}{\log(b/a)}\right] - \frac{1}{4\nu}\left[(r^2 - a^2) - (b^2 - a^2)\frac{\log(r/a)}{\log(b/a)}\right]\sum_{n=1}^{\infty}c_n(1 - nx)e^{-nt}\\ &= -\left[(r^2 - a^2) - (b^2 - a^2)\frac{\log(r/a)}{\log(b/a)}\right]\frac{1}{4\nu}\left[c_0 + \sum_{n=1}^{\infty}c_n e^{-nt} - x\sum_{n=1}^{\infty}nc_n e^{-nt}\right]\\ &= \frac{1}{4\nu}\left[(a^2 - r^2) + (b^2 - a^2)\frac{\log(r/a)}{\log(b/a)}\right]\left(1 + x\frac{\partial}{\partial t}\right)\left(-\frac{1}{\rho}\frac{\partial p}{\partial z}\right)\end{aligned}$$

When K is large, we have

$$J_0(Kr) \simeq \left(\frac{2}{\pi rK}\right)^{1/2}\cos\left(rK - \frac{\pi}{4}\right)$$

and

$$Y_0(Kr) \simeq \left(\frac{2}{\pi rK}\right)^{1/2}\sin\left(rK - \frac{\pi}{4}\right)$$

Substituting in (9), we have

$$w = \frac{c_0}{4\nu}\left[(a^2 - r^2) + (b^2 - a^2)\frac{\log(r/a)}{\log(b/a)}\right] + \sum_{n=1}^{\infty}\left[\left(\frac{a}{r}\right)^{1/2}\frac{\sin K(b - r)}{\sin K(b - a)} + \left(\frac{b}{r}\right)^{1/2}\frac{\sin K(r - a)}{\sin K(b - a)} - 1\right]\frac{c_n}{n}e^{-nt}$$

Reference

[1] Verma, P D , "On the flow of a viscous liquid under exponential pressure gradient superposed on the steady laminar motion of incompressible fluid between two co-axial cylinders," Proc Natl Inst Sci India **26** (1–12), 266 (1960)

A Load-Sinkage Equation for Lunar Soils

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Nomenclature

p = pressure, load divided by footing area, psi
z = sinkage, in
k_c, k_ϕ = soil-sinkage moduli, dimensions depend upon n
n = dimensionless exponent related to shape of pıessure-sinkage curve
b = smallest dimension of footing area, e g , width of track or tire
a_1, a_2 = soil parameters, coefficients of polynominal

Introduction

SOIL models ranging from hard rock to fine dust[1–4] have been proposed for the lunar surface Since the fine-dust hypothesis presents the greater challenge to vehicle mobility, it is being studied somewhat extensively and is the only type of soil considered herein

It is convenient to fit load-sinkage data with a mathematical equation in order that calculations can be made regarding the resistance of the soil to vehicle motion and the amount of energy absorbed by the soil in its deformation This energy should be allowed for in fuel consumption estimates and in powerplant selection In addition, the form of the equation establishes parameters that may be used in a dimensional analysis and model study

Since the load-sinkage equation now in most common use,[5] $p = (k /b + k_\phi)z^n$, was selected as a best fit to a wide variety of soils, presumably including dry granular soil, silt, loam, clay, etc , it was considered desirable to compare this equation with the data available for presumed lunar-type soil, namely, the dry granular type

The data considered included five soils (tuff, scoria, rhyolite, pumice, and basalt) selected by Chrysler geologist Milton Schloss as being representative of possible lunar soils Their particle size ranged from 53 μ up to 3 36 mm They were tested in atmosphere, in moderate vacuum, and in very high vacuum In addition, data available in the literature were considered These included vacuum tests of pumice[6] and of white silica flour[7] Finally, data from atmospheric testing of dry sand and gravel were considered

The parameters in the equation $p = (k /b + k_\phi)z^n$ are normally obtained by plotting pressure-sinkage data from two different size footings on log-log paper and fitting a straight line to each set of data The slope of each line determines the value of the exponent n The lines are, of course, parallel if the equation fits the data The k and k_ϕ moduli are then found by solving simultaneous equations relating the p-z intercepts and the footing size A straightforward application of this procedure often leads to negative values of k for dry soils[8] Since the k /b term can be made large compared to k_ϕ for sufficiently small b, e g , a narrow tire, this term can introduce a large error Therefore, most investigators set $k = 0$ when considering this type of soil This is equivalent to stating that the pressure-sinkage relationship is independent of footing size The problem here is that the random variation of the soil properties is larger than the size effect, so that any attempt at extrapolation of the size effect is dangerous For these reasons, no size effect will be considered in this paper, and comparison is made between the equation in the form $p = k_\phi z^n$ and a polynomial

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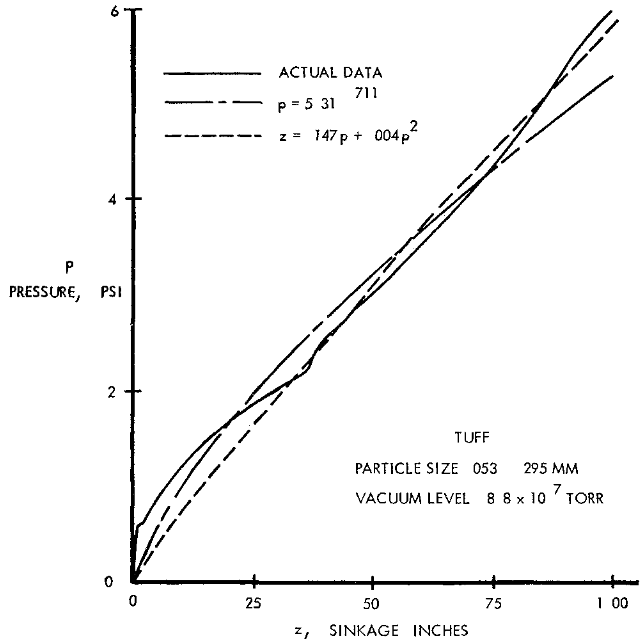

Fig. 1 Soil tests in moderate vacuum

By the Weierstrass approximation theorem,[9] it is known that any function continuous over some finite interval can be uniformly approximated by a polynomial of some degree to any desired accuracy The polynomial is then a useful approximation as long as the number of terms necessary to obtain the desired accuracy is not excessive In the following, a comparison is made between the power function $p = k_\phi z^n$ and polynomials of degree ranging from one to seven

Analysis

A digital computer program has been written to calculate the best (in a least-squares sense) values for the coefficients for each of the seven polynomials considered A print-out is made of the coefficients and the root-mean-square difference between the data points and the polynomials A similar routine is followed for the equation $p = k_\phi z^n$, except that the transformation $x = \log p$ and $y = \log z$ is made, which is equivalent to plotting the data on log-log paper and fitting a straight line to the data thus plotted It should be noted that this transformation compresses the data at high values, in exactly the same manner as a plot of the data on log-log paper This amounts to a bias, or weighting, of the low values of pressure and sinkage

Results

A total of 51 sets of soil data was considered In every case, a third-degree polynomial $z = a_1p + a_2p^2 + a_3p^3$ gave